Systemic Credit Freezes in Networks

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Roadmap

1. Introduction

2. Unsecured Lending Networks
   - Model and Equilibrium
   - Existence, Uniqueness, and Robustness
   - Comparative Statics in Intermediation Chains
   - Characterization of Credit Freeze
   - Policy Analysis

3. Collateralized Repo Networks
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   - Worked Example
   - Future Work
Motivation

- By the onset of the financial crisis of 2008, the US financial system had become increasingly more interconnected.
  - Complex lending relations: interbank and overnight lending, securitized lending such as repo market.
- Failure of an institution may trigger financial distress for its counterparties or those holding its shares.
- Lenders need to also assess creditworthiness of borrowers of the borrower, and so on.
- Collapse of Lehman Brothers in September 2008 causes many institutions to lose access to credit (credit freeze).
Ex-Ante vs. Ex-Post

- **Ex-Post Contagion:** the failure of one institution can cause other institutions to fail.
- **Ex-Ante Considerations:** credit freezes induced by the fear future illiquidity. Profitability of borrowers might be compromised because of ex-post effects.

“You have a neighbor, who smokes in bed. . . Suppose he sets fire to his house. You might say to yourself. . . ‘I’m not gonna call the fire department. Let his house burn down. It’s fine with me.’ But then, of course, what if your house is made of wood? And it’s right next door to his house? What if the whole town is made of wood?”

Ben Bernanke  
Chair of Federal Reserve Bank  
during the 2008 financial crisis

“...Institutions such as Goldman Sachs, Credit Suisse and Deutsche Bank had “little or no interest to renew repos [for Bear Stearns] in the face of concerns over the dealer bank’s solvency.”

Darrell Duffie  
*How Big Banks Fail and What to do About It*  
March 27, 2010

“If we start taking novations [credit contracts for Bear Stearns], people pull their business, they pull their collateral, you’re out of business.”

Gary Cohn  
Co-President  
Goldman Sachs  
March 11, 2008
Ex-Post Analysis

- Basic setup: \( n \) banks, survival of bank \( i \) depends on both (1) an idiosyncratic shock at \( i \), and (2) the survival of other banks.

- We model the dependence structure in (2) using a financial network \( G_\ast \):

- Dependence may capture unsecured debt contracts, collateralized lending, cross-company share holdings, among others.

- Main point: A negative shock can spread to the rest of the network, causing systemic trouble.

What We Do

- A bank-level model of financial intermediation
  - ex-ante incentives of the banks to make profitable loans
  - endogenous lending contracts and financial network
  - endogenous risk and defaults

- Banks' fear of future default determines interconnectivity of financial lending.
- Credit freezes arise for small changes to risk in the network.

- Today:
  - existence and uniqueness results
  - comparative statics
  - characterization of credit freezes
  - policy implications
Overview

(a) *Systemic Credit Freezes in Financial Lending Networks* (complete)

- **Unsecured** debt contracts intermediate between depositors (cash lenders) and entrepreneurs (cash borrowers).
- Uncertainty about banks’ future liquidity and *ability to repay loan*.
- Profitability of loans depends on network of lending and anticipation of future liquidity problems.

(b) *Systemic Credit Freezes in Repo Networks* (preliminary)

- **Secured** lending contracts intermediate between money market funds (cash lenders) and hedge funds (collateral providers).
- Secured lenders are better protected against default risk, but if the lender defaults, may be impossible to recover the collateral: leads to a *settlement failure*.
- How does the sensitivity of lending (and credit freeze) change if lenders can demand collateral for lending?
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Talk on Financial Lending Networks

- More importantly, banks fear future liquidity problems ex-ante, leads to systemic credit freeze.

- We develop a stylized model of ex-ante credit freezes in a financial network:
  - Banks have outside known liabilities (e.g., employee wages, operational costs) and also hold assets with random value.
  - Some banks can lend to clients located at the leaves of the network which a fixed demand for funds.
  - Lending contracts determined by potential lenders who offer an interest rate and borrowers decide to borrow as much as desired.
  - Potential lenders can always freeze credit by offering no contract and avoiding any subsequent losses.

- Introduce risk shifts that increase the likelihood and severity of future liquidity issues for certain banks in the network.

- Characterize the subgame perfect equilibria of this financial network.
Main Results

- **Properties** of the equilibrium:
  - Existence of a pure strategy equilibrium, and uniqueness of a stronger equilibrium notion.
  - Financial network is always a directed-tree between ultimate cash lenders and borrowers.

- **Comparative statics** for the chain network. Freeze occurs when:
  - Many layers of financial intermediation or liquidity mismatch is small.
  - Asset markets are weak and/or unstable.
  - Portfolios of assets across banks are independent or anti-correlated.

- In **tree networks**, where each bank can borrow from at most one other bank, freezes are “simple” in the sense that:
  1. They always originate with the affected bank (the bank receiving the shock).
  2. The set of banks experiencing a credit freeze is a connected set.
Main Results, cont.

- In **general networks**, a negative shock can affect the equilibrium in nuanced ways and freezes may “complex.”
  - **Non-monotone**: increase in the risk of some bank $i$ leads to increase in lending.
  - Two types of **complexity**: (i) bank with increased risk does not lose credit but some other bank does, and (ii) increase in risk of one part of network causes some other distinct segment of the network to lose credit.

- Because systemic credit freeze can occur from a small, isolated shock to risk, (relatively) **inexpensive policy** can restore large amounts of lending.
Related Literature

• Empirical evidence of credit freezes in interbank lending
  ▶ Adrian et al. (2013); Alfonso, Kovner and Schoar (2010); Brunnermeier (2009)

• Endogenous network formation
  ▶ Leitner (2004); Babus (2006); Blume et al. (2011)

• Single bank or pair of banks accessing credit market
  ▶ Gorton and Metrik (2012); Diamond and Rajan (2011); Caballero and Simsek (2013)

• Ex-ante fears captured through coordination game
  ▶ Allen and Babus (2009); Anand et al. (2012); building off global games literature of Shin and Morris (2001)
  ▶ No ex-post trigger

• To the best of our knowledge none of this literature studies ex-ante credit freezes in financial networks.
Banks, Depositors, and Entrepreneurs

(a) **Entrepreneur** ($\mathcal{E}$): Non-financial “bulky” project with return $r^*$ for one unit of investment ($1$).

(b) **Depositor** ($\mathcal{D} = \{0\}$): Competitive market of depositors with access to outside risk-free technology with return $r_0$.

(c) **Bank** ($\mathcal{B} = \{1, \ldots, n\}$): Intermediaries between depositors and entrepreneurs, and each other.

![Opportunity Network](image)

**Figure:** Opportunity Network $G$. 

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Timing of Interbank Lending

- Take directed, opportunity network $G$ as given. Let $N_{in}(i)$ and $N_{out}(i)$ denote the in and out-neighborhood of $i$, respectively.

- Lending model consists of three stages:

  - **Offer:** Banks sequentially make interest rate offers $R_{i \rightarrow j}$
  - **Borrow:** Banks sequentially decide to borrow $x_{i \rightarrow j}$ at $R_{i \rightarrow j}$
  - **Repayment:** Banks make repayments on loans after realizing asset values
Repayment Equilibrium

- Take financial network $G_\ast = (R, x)$ as given.
- The (realized) profit of bank $j$ is

$$\pi_j = z_j + \sum_{k \in N_{out}(j)} y_{k \rightarrow j} - \sum_{i \in N_{in}(j)} R_{i \rightarrow j} x_{i \rightarrow j}$$

- If $\pi_j \geq 0$, the bank is *solvent* and makes full repayment on all its loans, $y_{j \rightarrow i} = R_{i \rightarrow j} x_{i \rightarrow j}$.
- If a bank defaults, it repays *nothing*. This is known as the *total failure* model, where bankruptcy liquidation proceeds are zero.
Lending Equilibrium

• Every bank $j$ maximizes expected upside profit minus a default cost ($F \geq 0$) from bankruptcy, $\mathbb{E}[(\pi_j)^+ - F \cdot d_j]$, subject to the borrowing constraint:

$$\sum_{i \in \mathcal{N}_{in}(j)} x_{i \rightarrow j} \geq \sum_{k \in \mathcal{N}_{out}(j)} x_{j \rightarrow k}$$

• Weak solution concept: subgame perfect equilibria.

• Strong solution concept: refine subgame perfection to eliminate indifferences; trembling-hand perfect equilibrium for interest rate offers.

• Essential uniqueness: two financial networks $G_*, G'_*$ are equivalent if $x = x'$ and $R_{i \rightarrow j} = R'_{i \rightarrow j}$ agree wherever $x_{i \rightarrow j} > 0$. 
Strong Equilibrium Properties

Theorem

For any opportunity network $G$:

(i) There exists a strong lending equilibrium in pure strategies.

(ii) For a generic probability distribution over $z$, the strong lending equilibrium is essentially unique.

(iii) Financial network $G_*$ is a directed tree.

Figure: Opportunity Network $G$ (dashed) and Financial Network $G_*$ (solid).
Sensitivity to Order

Theorem

For any $G$, the borrowing network $x$ does not depend on the sequential order of interest rate offers or the sequential order of lending/borrowing.

- Can the interest rate network ($R$) depend on the order?
Intermediation Chain Example

- Single depositor 0, single client \( m + 1 \).
- Every bank has iid returns \( z_i \in \{-\infty, \sigma\} \) where \( \sigma \in (0, 1) \) and return \( z_i = \sigma \) occurs with probability \( p_i = 1 - \varepsilon \) for small \( \varepsilon \).
- Increasing risk premia as you move up the chain because of greater default risk:

\[
R_{(m-k)\to(m-k+1)} - R_{(m-k-1)\to(m-k)} = \frac{1 - p^k}{p^k} \sigma \approx k\varepsilon\sigma
\]

- Interest rates in equilibrium given by \( R_{(m-k-1)\to(m-k)} \approx r^* - \frac{1}{2} k^2 \varepsilon \sigma \).

  - Can only support \( \approx \beta \sqrt{(r^* - r)} \) banks without a credit freeze (for some constant \( \beta \)).
Intermediation and Liquidity Mismatch

- We say that bank \( j \) has a **credit freeze** if \( R_{i \rightarrow j} = \emptyset \) for all \( i \in \mathcal{N}_{in}(j) \) for some equilibrium financial network \( G_* \).

- If \( G \) is a chain, then every bank has a credit freeze or no bank does.

**Theorem**

(a) **Length**: There exists threshold \( M \) number of banks such that for \( m < M \) there is no freeze and for \( m > M \) the whole chain freezes (regardless of risk).

(b) **Liquidity Mismatch**: There exists threshold \( s^* \) such that for \( (r^* - r_0) > s^* \) there is no freeze and for \( (r^* - r_0) < s^* \) the whole chain freezes.

- As banks face possible **ex-post contagion** from downstream defaults, will only lend if loans are sufficiently **profitable ex-ante**.
  - Gains from trade are fixed, so added risk can lead to credit freeze.
Shocks to Asset Distribution

Definition

Say that \( z' \) first-order stochastic dominates \( z \) if \( z'_i \mid z'_\sim i \) FOSD \( z_i \mid z_\sim i \) for all banks \( i \) and all realization \( z_\sim i \).

- **Stronger condition**: In every state of the world, asset \( z' \) pays more than \( z \) for all banks.

Theorem

exists \( \tilde{F} > 0 \) such that for all \( F > \tilde{F} \), whenever \( z' \) FOSD \( z \), there is no systemic credit freeze in \( z' \) if there is no systemic freeze in \( z \).

- Negative shocks to the distribution of asset returns cause freezes.
- Two competing effects: **systemic risk** and **risk appetite**.
  - Require that \( F \) be sufficiently large to ensure fear of risk dominates change in risk appetite.
- Similar result for a special case of second-order stochastic dominance (see paper).
Portfolio Correlation

- For simplicity, assume asset returns are normally distributed with mean $\mu > 0$, variance $\sigma^2$, and correlation $\rho$.

**Theorem**

For a fixed chain network $G$, there exists $\rho^* < 1$ such that if $\rho > \rho^*$ there is no credit freeze.

- As $\rho \to 1$, lending becomes “riskless” because all banks default in the same state of the world.

- As returns become more independent (or anti-correlated), bank $i$ gets a positive return when some other bank might default, which makes lending riskier.
Simple Freezes in Trees

- Beyond intermediation chains...
- **Risk shift**: parallel change in the distribution of $z_i$, set $z'_i = z_i - \epsilon$ for some shock size $\epsilon > 0$.
- **Simple freeze**:
  1. Bank $i$ loses credit
  2. All banks which lose credit are connected to bank $i$ through banks also with frozen credit.

**Proposition**

*If $G$ is a directed tree, and default cost $F$ is not too large, then any (idiosyncratic) risk shift induces only simple freezes.*
Non-Monotonicity: Before Risk Shift

- Bank 3 has unique access to a big project, bank 4 has unique access to a small project, and banks 3 and 4 compete over another project.
- Bank 3 has lower risk of default than bank 4.
- Intermediation chain $2 \rightarrow 4$ cannot compete with $1 \rightarrow 3$ because of added risk.
- Bank 2 may find the $1$ loan to bank 4 unprofitable given default risk of bank 4.
  - Entrepreneur 3 has a credit freeze.

**Figure**: Before Shift.
Non-Monotonicity: After Risk Shift

• Bank 3 is now riskier than bank 4. Intermediation chain 1 → 3 cannot compete with 2 → 4 because bank 1 must demand a higher interest rate.

• Chain 2 → 4 now has two projects, and bank 2 makes positive expected profits on a loan of $2.

• Entrepreneur 3 gains access to credit following increased risk at bank 3.
  ▶ No other entrepreneur loses access to credit.

Figure: After Shift.
Complex Freezes: Before Risk Shift

- Each bank has independent returns: \textbf{G(ood)} or \textbf{B(ad)}
- \textbf{B}: toxic asset wipes the bank out
- Green banks are always safe (realize state \textbf{G} with probability 1)
- Small chance yellow and pink banks get \textbf{B} return. Assume pink bank is slightly riskier.
- Branch A to \textbf{E}_2 is riskless so is more competitive than branch B.
- Branch C has two clients as opposed to one, so branch C can compete with branch B over \textbf{E}_3.

\textbf{Figure}: Before Shift.
Complex Freezes: After Risk Shift

- Shift risk of bank in branch A (red): realizes state B with probability 1 (for simplicity).
- Clearly branch A will not lend to \( E_2 \), so branch B has monopolistic access over \( E_2 \).
- Bank 3 is less risky than bank 5, and both branch B and C have access to two clients.
- Branch B is now competitive for client 3, so branch C can only have access to \( E_4 \).
- Profits from client \( E_4 \) not sufficient to compensate for pink bank’s risk.
  - Entrepreneur 4 loses access to credit, despite a shock to a separate part of the network.

Figure: After Shift.
Central Bank Policy

- Central bank has a budget $B$ and can implement **asset purchases** (positive risk shifts) or **discount window** (lender-of-last-resort) policies.
  - Asset purchases: shift distribution of $z_i$ by $\epsilon_i$.
  - Discount window: offer to lend $B_i$ to bank $i$ at favorable rate.

- Space of feasible policies $(\epsilon, B)$ at discount rate $r_{CB} > 1$:
  \[
  \sum_{i=1}^{n} B_i \leq \bar{B} \quad \text{and} \quad \sum_{i=1}^{n} \epsilon_i \leq \bar{\epsilon}
  \]

- **Untargeted** policy: Use the entire budget and set $\epsilon_i = \bar{\epsilon} / n$ and $B_i = \bar{B} / n$ for all banks.

- **Targeted** policy: No restriction on $(\epsilon, B)$ except the budget constraint.

- Optimal policy: maximize total lending to entrepreneurs.
Main Policy Findings

• For freezes in the chain, an untargeted policy does no worse than a targeted policy.
  ▶ Lending in the chain is cooperative, use the interest rates to redistribute liquidity in a way that makes everyone content to borrow/lend.

• After a risk shift, we see entrepreneur lending decrease by $\Delta > 0$ from a simple freeze, want to implement an inexpensive policy to restore lending:
  ▶ Always set $\epsilon_i = 0$ and $B_i = 0$ if bank $i$ did not lose credit.
  ▶ This policy is relatively cheap (i.e., $B \leq \Delta$) and strictly cheaper than untargeted policy (if not in the chain).

• When the freeze is complex, may be better to target banks unaffected by freezes.
Other Policy Features

• An untargeted policy will work with a large budget, but if the resources of the central bank are limited (i.e., the budget is bounded), could cause entrepreneur lending to fall.
  ▶ Increasing the profitability of all lending paths might not relieve competition effects that cause credit freezes.

• If policymakers are misinformed of the financial network, targeting policies can exacerbate the problem.
  ▶ A rescue policy that only targets banks in distress can lead to more credit freezes because of network effects.

• Evidence for a policy of decreasing some asset prices, which would lower interest rates.
Conclusion

- Extend current work on financial networks: link between ex-post defaults and ex-ante lending considerations.

- Lack of short-term funding because of uncertainty of future solvency:
  - Bear Stearns was in trouble (March 2008) months before the collapse of Lehman Brothers (September 2008).
  - Interconnectedness of financial system caused tightening of credit. Affected large financial institutions and small business alike.

- Extent of credit freeze is highly sensitive to the structure of lending.

- Rescue policy can be effective if the cause of the freezes is well-understood. Policy becomes increasingly more complex as financial system becomes more complex.
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Why consider repo networks?

- In 2008 financial crisis, short-term funding through repurchase agreements played a key role in the failure of many institutions.
  - Total repo lending at the peak of the crisis exceeded $4 trillion USD, but fell by almost 50% by 2009 (Krishnamurthy et al. (2014)).

- Three types of agents: cash lenders, cash borrowers, and intermediaries.
  - Cash lenders (typically money market funds) are cash-rich, invest in intermediaries who pay interest in the tri-party repo market.
  - Cash borrowers (typically, hedge funds) need cash for projects, hold only illiquid assets on balance sheet.
  - Intermediaries trade with cash borrowers and each other in bilateral repo market.

- If the cash borrower posts illiquid assets as collateral, then lender is less exposed to direct counterparty risk.
  - Reduces the likelihood of credit freeze and provides more liquidity to financial system.
  - Do any new vulnerabilities emerge?


Repayment Risks

- **Cash lender**: the cash lender provides cash and in exchange receives securities (collateral) at time $t = 1$. Between time $t = 1$ and $t = 2$:
  
  (a) The cash **borrower** may become insolvent and unable to repay the loan, so the **lender** gets to keep the borrower’s collateral.
  
  (b) However, the **lender** is exposed to price fluctuations of the collateral, which may or may not cover the principal of the loan.
  
  (c) This may make the cash lender insolvent, inducing a **default cascade**.

- **Cash borrower**: the cash borrower provides securities (collateral) and in exchange receives cash at $t = 1$. Between $t = 1$ and $t = 2$:

  (a) The cash **lender** may be insolvent and unable to return the collateral tied up in bankruptcy. This induces a settlement failure, where the **borrower** keeps the cash but loses its collateral.
  
  (b) If the collateral had been reused for another collateralized loan, then the **borrower** may be unable to secure the assets needed to unwind other repo positions.
  
  (c) Bank may be unwilling or unable to return rehypothecated collateral, triggering a **settlement failure cascade**.
  
  (d) **Iyer and Macchiavelli (2017)** show that in near 100% of settlement failure cases, banks do not use liquidity to buy rehypothecated collateral from the market, but instead fail to settle themselves.
Model and Assumptions

- Three period model $t = 0, 1, 2$.

- Cash lenders offer contracts sequentially at $t = 0$ which specify repo rate $R$, haircut $h$, and cash amount $x$. Cash borrowers either accept or reject at $t = 1$.

- Hedge funds are endowed with illiquid collateral and MMFs are endowed with cash; banks act as intermediaries.

- At $t = 2$, collateral value $\tilde{a}$ and liquidity shocks $z_i$ are realized; agents make repayments of loan and settle collateral wherever possible.

- Illiquid collateral may be converted into cash and vice-versa, at a cost.
  - If a bank $j$ cannot repay cash loan, bank $j$ defaults, loses all cash and assets, and receives payoff 0.
  - If bank $j$’s collateral is tied up in a settlement failure, bank $j$ can either purchase illiquid securities from market or fail to settle as well.
Illustration of Repo: Normal Times vs. In-Default
Example with Intermediation Chain

- Recall the $m$-bank chain where each bank has asset return $z_i \in \{-\infty, \sigma\}$, $z_i = \sigma$ with probability $p = 1 - \varepsilon$, and we replace the depositor with a money market fund (MMF) and the entrepreneur with a hedge fund (HF).

- The hedge fund owns an indivisible illiquid asset $\tilde{a}$, which when liquidated at $t = 2$ has random value on support $[a, \bar{a}]$ with $a \geq r_0$.

- What happens if all loans are collateralized by the $\tilde{a}$ asset?
  - If some bank $j$ realizes $z_j = -\infty$, it defaults. Let $j^*$ be the bank closest to MMF which defaults. Bank $j^* - 1$ keeps the asset and all banks $k > j$ keep the $1$ loan.

- Because the liquidation value of the asset always exceeds interest payment, no systemic default risk.
  - Implies that for any $r_0 < r^*$, there is no credit freeze.
  - Collateral substitutes for risk premium needed to compensate for credit risk.
Fear of Settlement Failure

• In reality, the hedge fund’s expected value of the asset is greater than that of other financial agents in the network.
  ▶ Belief Disagreement: Hedge funds seek leverage and hold optimistic views about their assets (see Simsek (2013) and Infante (2015)).
  ▶ Fire Sales: Safe harbor provisions and legal requirements might force seized collateral to be liquidated quickly and inefficiently (see DiMaggio and Tahbaz-Salehi (2014) and Morrison et al. (2014)).

• For simplicity, assume that the hedge’s fund collateral is worth face value, whereas seized collateral recovers only $\alpha \leq 1$.

• When $\mathbb{E}[\tilde{a}] > 1$, the hedge fund posts collateral worth more than the loan. If some bank $j$ defaults, the collateral is seized by $j$’s lender and begins a cascade of settlement failures ending with the hedge fund losing its collateral.
Collateralized Credit Freezes

- Probability that some bank $j$ defaults is equal to $1 - p^m$. If $R$ is the repo rate charged to entrepreneur $E_1$, then the expected value of receiving the collateralized loan from bank 1 is:

$$\pi_{E_1} = -(1 - p^m)(1 - \mathbb{E}[\tilde{a}]) + p^m(r^* - R)$$

- Expected value to cash borrower is positive if and only if:

$$m \leq \log \left(1 + \frac{r^* - R}{\mathbb{E}[\tilde{a}] - 1}\right) \frac{\log(1/p)}{\log(1/p)}$$

- If $\alpha$ is not too small, then interest rates can be identical to $r_0$, so there is no credit freeze in secured lending if and only if:

$$m \leq \log \left(1 + \frac{r^* - r_0}{\mathbb{E}[\tilde{a}] - 1}\right) \frac{\log(1/p)}{\log(1/p)}$$

- Collateralized credit freeze occurs when cash borrower withdraws collateral before the realization of liquidity shocks.
Cash Run or Collateral Run on Repo?

- **Cash-driven runs** (Gorton and Metrick (2012)) and collateral-driven runs (Infante and Vardoulakis (2019)) on repo. Normalize $\alpha \mathbb{E}[\tilde{a}] = r_0 = 1$.

- Binding constraints of a credit freeze in collateralized lending:
  1. All banks and MMF must be willing to lend given the **default risk** of downstream borrowers.
  2. Hedge fund must be willing to borrow given **settlement risk** of losing collateral.

- Banks are unwilling to provide unsecuritized loans unless:
  \[ m \leq \sqrt{\frac{r^* - r_0}{2\varepsilon \sigma}} \]

- Hedge funds unwilling to provide collateral unless:
  \[ m \leq \frac{\alpha (r^* - r_0)}{\varepsilon (\alpha (r^* - r_0) + (1 - \alpha))} \]
Limits to Rehypothecation

- Suppose that collateral is rehypothecated for only $k < m$ banks in the chain, then the remainder of the chain is financed by unsecured lending.
  - Can capping rehypothecation in lending chains help with credit freeze?

Proposition

Let $m_u^*$ and $m_s^*$ be the lengths of the longest chains in which there is no credit freeze in the unsecured and fully secured lending networks. Then there exist collateralized lending contracts such that there is no credit freeze in the chain with $m_u^* + m_s^*$ banks.

- Allow reuse of hedge fund’s collateral until bank $m_s^*$, who holds the collateral. All other loans from MMF are unsecured.
Going Forward

• What happens if banks charge different haircuts (i.e., hoard some of the asset for excess liquidity)?
  ▶ Can be used to insulate banks against borrower defaults, but may make others more susceptible.

• If $t = 2$ price of asset $\tilde{a}$ is random, then lender who recovers the collateral might still default on the cash loan if $\tilde{a}$ is realized low.
  ▶ In a fully collateralized chain, if bank $j$ is the bank hit with the liquidity shock, whether the shock spreads to bank $j$’s lenders depends only the realization of the asset.

• In more complex networks (i.e., multiple borrowers and lenders), settlement failures may not always cascade.
  ▶ If a bank’s lender fails to return the collateral, but a bank’s borrower defaults, it may seize that collateral and use it unwind a repo with a different borrower.
Key Takeaways

- Collateralization of the loan can reduce credit freezes and generate additional liquidity.
  - Fear of future defaults are mitigated because the lender is protected.
  - Liquidation of seized assets can lead to settlement failures of the repurchase agreement, inducing fear for collateral providers.

- Changes to the quality of the collateral, gains from trade, or network risk can create credit freezes for different reasons.
  - Hedge funds may refuse to post collateral or lenders may refuse to lend out cash.

- Network position of risk shifts impact likelihood of freeze differently in securitized and unsecuritized lending.
  - **Securitized lending**: freezes occur from changes in risk to banks nearer money market funds.
  - **Unsecured lending**: freezes occur from changes in risk to banks nearer hedge funds.