Propagation of Credit Freezes in Lending Networks

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INFORMS

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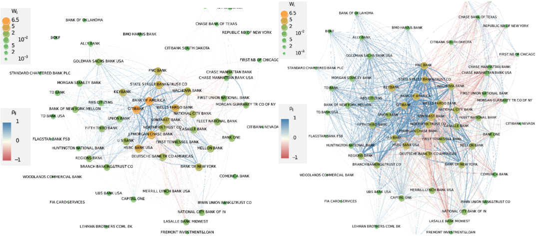
Motivation

- By the onset of the financial crisis of 2008, the US financial system had become increasingly more interconnected.
  - Complex lending relations: interbank and overnight lending, securitized lending such as repo market.
- Failure of an institution triggers financial distress for its counterparties or those holdings its shares.
- Lenders need to also assess creditworthiness of the borrowers of the borrower, and so on.



- Collapse of Lehman Brothers in September 2008 causes many institutions to lose access to credit (credit freeze).

Ex-Post vs. Ex-Ante

- Ex-Post Contagion: the failure of one institution can cause other institutions to fail.
- Ex-Ante Considerations: credit freezes induced by the fear future liquidity or profitability of borrowers might be compromised because of ex-post effects.

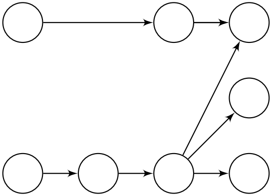
> "You have a neighbor, who smokes in bed...Suppose he sets fire to his house. You might say to yourself...'I'm not gonna call the fire department. Let his house burn down. It's fine with me.' But then, of course, what if your house is made of wood? And it's right next door to his house? What if the whole town is made of wood?"
>
> Ben Bernanke,
> Chair of Federal Reserve Bank

> Institutions such as Goldman Sachs, Credit Suisse and Deutsche Bank had "little or no interest to renew repos [for Bear Stearns] in the face of concerns over the dealer bank's solvency."
>
> Darrell Duffie,
> *How Big Banks Fail And What to do About It*,
> March 27, 2010

> "If we start taking novations [credit contracts for Bear Stearns], people pull their business, they pull their collateral, you're out of business."
>
> Gary Cohn,
> Co-President,
> Goldman Sachs,
> March 11, 2008

Ex-Post Analysis

- Basic setup: n banks, survival of bank i *depends* on both (1) an idiosyncratic shock at i, and (2) the survival of other banks.
- We model the dependence structure in (2) using a network T:



- Main point: A single negative shock can to the rest of the network, causing systemic trouble.
- Studied extensively in previous literature: Acemoglu, Ozdaglar and Tahbaz-Salehi (2015), Cabrales, Gale and Gottardi (2015), Elliott, Golub and Jackson (2014), Gai and Kapadia (2010), Jorian and Zhang (2010)

This Talk

- More importantly, banks fear future liquidity problems ex-ante, leads to systemic credit freeze.
- We develop a stylized model of ex-ante credit freezes in a financial network:
  - Banks have outside known liabilities (e.g., employee wages, operational costs) and also hold assets with random value.
  - Some banks can lend to clients located at the leaves of the network which have a fixed demand for funds.
  - Lending contracts determined by potential lenders who offer an interest rate and borrowers decide to borrow as much as desired.
  - Potential lenders can always freeze credit by offering no contract and avoiding any subsequent losses.
- Introduce shocks to equity value that increase a bank's probability of default.
- Characterize the subgame perfect equilibria of this financial network.

Main Results

- Lending networks with many layers of intermediation and unstable asset markets are most susceptible to credit freeze.
- In tree networks, where each bank can borrow from at most one other bank, freezes are "regular" in the sense that:
  1. They always originate with the affected bank (the bank receiving the shock).
  2. The set of banks experiencing a credit freeze is a connected set.
  3. The size of the credit freeze (the number of banks affected) is monotonically increasing in the size of the initial shock.
- In general networks, a negative shock can affect the equilibrium in complex ways and freezes may be "irregular."
- Because systemic credit freeze can occur from a small, isolated shock to risk, (relatively) inexpensive policy can restore large amounts of lending.

Related Literature

- Empirical evidence of credit freezes in interbank lending:
  - Adrian et al. (2013); Alfonso, Kovner and Schoar (2010); Brunnermeier (2009)
- Network formation:
  - Leitner (2004); Babus (2007); Blume et al. (2011)
- Single bank or pair of banks accessing credit market:
  - Gorton and Metrik (2012); Diamond-Rajan (2011); Caballero-Simsek (2013)
- Ex-ante fears captured through coordination game:
  - Allen et al. (2009); Anand (2011); building off global games literature of Shin-Morris (2008)
  - No ex-post trigger

Banks, Depositors, and Clients

1. Client project ($\mathcal{C}$): Non-financial project with funding level x and capacity $\bar{x}$ has production technology:

$$f(x) = \begin{cases} r^*, & \text{if } x \geq \bar{x} \\ 0, & \text{otherwise} \end{cases}$$

   for $r^* > \bar{x}$.

2. Depositor ($\mathcal{D}$): Perfectly elastic supply of funds at interest rate r.

3. Bank ($\mathcal{B}$): Intermediaries between depositors and clients (and each other).

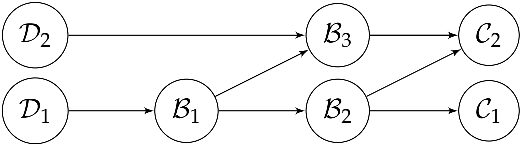


Figure: Potential Lending Network T.

Ex-Ante Lending Game

- Take the directed, potential lending network T as given (assume acyclic). Let $\mathcal{N}_{in}(i)$ and $\mathcal{N}_{out}(i)$ denote the in and out-neighborhood of i, respectively.

- Two-stage game:

  1. Offer Stage: Banks make offers sequentially according to a deterministic order. At every time t, some bank i makes take-it-or-leave-it offer $R_{i\to j} \in \mathbb{R}_+ \cup \{\infty\}$ to every bank $j \in \mathcal{N}_{out}(i)$. We can generalize this to simultaneous offers.

  2. Borrowing Stage: Each bank j simultaneously chooses to borrow $x_{i\to j}$ at interest rate $R_{i\to j}$ for every $i \in \mathcal{N}_{in}(j)$.

Ex-Post Repayment Game

- Each bank i has an outside balance sheet:
  - $v > 0$: outside, fixed liabilities
  - $\eta_i > 0$: random return of outside asset, drawn from joint distribution H
  - $z_i = \eta_i - v$: the random outside equity value of bank i
- The ex-post (endogenous) variables of the network are:
  - d_j: the event that bank j defaults (binary)
  - $y_{j \to i}$: the repayment of j to i
- A (deterministic) ex-ante equilibrium consists of the pair, $(\mathbf{R}, \mathbf{x})$, specifying interest rates and borrowed funds in the network.
- A (random) ex-post equilibrium consists of the pair, $(\mathbf{d}, \mathbf{y})$, specifying defaults and repayments, conditional on the realizations of $\mathbf{z}$.

Ex-Post Equilibrium Concept

- The (realized) profit of bank j is

$$\pi_j = z_j + \sum_{k \in \mathcal{N}_{out}(j)} y_{k \to j} - \sum_{i \in \mathcal{N}_{in}(j)} R_{i \to j} x_{i \to j}$$

- The default vector $\mathbf{d}$ satisfies $d_i = 1$ if and only if $\pi_i < 0$. The repayment vector $\mathbf{y}$ satisfies

$$y_{j \to i} = \begin{cases} R_{i \to j} x_{i \to j}, \text{ if } d_j = 0 \\ 0, \text{ if } d_j = 1 \end{cases}$$

for all $(i \to j) \in T$.

- If a bank defaults, it repays nothing. This is the total failure model, as compared to prorata model (ongoing work).

Ex-Ante Equilibrium Concept

- Every bank j maximizes expected upside profit, $\mathbb{E}[\pi_{j_+}]$, subject to the borrowing constraint:

$$\sum_{i \in \mathcal{N}_{in}(j)} x_{i \to j} \geq \sum_{k \in \mathcal{N}_{out}(j)} x_{j \to k}$$

- For this talk, we assume that z_i are iid according to:

$$z_i = \begin{cases} t - \sigma, \text{ w.p. } p_i \\ t + \frac{p_i \sigma}{1 - p_i}, \text{ w.p. } 1 - p_i \end{cases}$$

  - $t > 0$: mean outside equity value
  - $\sigma > t$: the level of "instability" of the asset's value
  - p_i: the probability the asset underperforms

- Solution concept: subgame perfect equilibria of the ex-ante game.

Ex-Post Equilibrium Existence and Uniqueness

Theorem (Ex-Post)

For every network T and every $(\mathbf{R}, \mathbf{x})$ there exists a unique ex-post equilibrium $(\mathbf{d}, \mathbf{y})$ for every realization of $\mathbf{z}$.

Proof Idea

1. Call $d_{i,\mathcal{C}}$ the max distance between bank i and any client $j \in \mathcal{C}$.
2. Consider $d_{i,\mathcal{C}} = 1$; compute π_i, d_i, and $y_{i \to j}$ for all $j \in \mathcal{N}_{in}(i)$.
3. Induct on $d_{i,\mathcal{C}}$.

Ex-Ante Equilibrium Existence

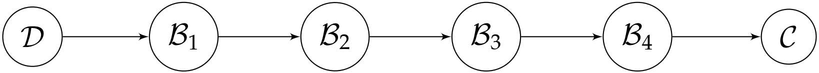
Theorem (Ex-Ante)

For every network T, there exists an ex-ante equilibrium $(\mathbf{R}, \mathbf{x})$ in pure strategies.

Proof Idea

1. Kakutani: exists an equilibrium in possibly mixed strategies.
2. Sequential game of complete information with equilibrium $\Longrightarrow$ equilibrium in pure strategies.
3. Extend argument to where banks take expectations over induced ex-post equilibrium conditional on action.

The Chain: Length



Definition (Freeze)

We say there is a freeze in the network T at bank j if there exists an ex-ante equilibrium where $R_{i \to j} = \infty$ for all $i \in \mathcal{N}_{in}(j)$.

Proposition

In the chain of length N*, every bank freezes or no bank does. There exists* $\bar{\mathrm{N}}$ *such that if* $\mathrm{N} > \bar{\mathrm{N}}$ *there is a freeze and if* $\mathrm{N} < \bar{\mathrm{N}}$ *there is no freeze.*

Volatility Comparative Statics

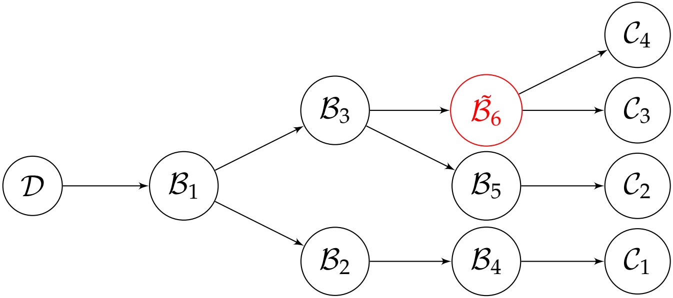
Proposition 2.

For fixed p, there exists $\bar{\sigma}_p$ such that if $\sigma > \bar{\sigma}_p$, the chain freezes and if $\sigma < \bar{\sigma}_p$ it does not. There also exists $\underline{p} < 1$ and $\bar{\bar{\sigma}} < \bar{\sigma}_{\underline{p}}$ such that if $\sigma < \bar{\bar{\sigma}}$ or $p > \underline{p}$, there is no ex-post contagion in equilibrium with probability 1.

- Intuition 1: Option-value of just holding the asset increasing in σ. Harder to meet IR constraints with possibility of ex-post spread.
- Intuition 2: Low enough σ or high enough p incentivizes banks to make sure there is no systemic risk.
- When $\sigma \in (\bar{\bar{\sigma}}, \bar{\sigma}_{\underline{p}})$ and $p < \underline{p}$, there is a moral hazard (MH) problem in the interbank market.
- $\underline{p}$ typically pretty small. 2-bank chain then $\underline{p} = 1/2$; 3-bank chain then $\underline{p} = (3 - \sqrt{5})/2 \approx 0.38$. Heavy tails cause MH.

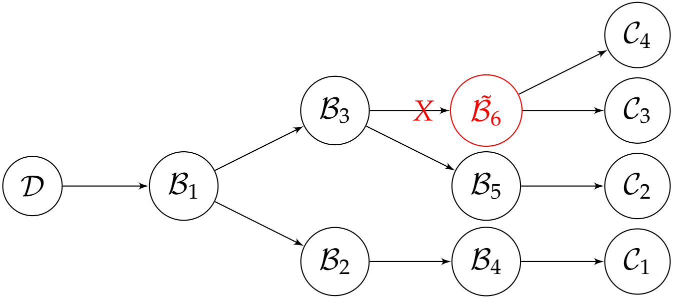
Trees

- Banks do not compete for contracts.
- *Shock*: Increase the likelihood of a bad return for bank 6 (i.e., $p'_6 > p_6$).
- There exist $t, \sigma, \{p_i\}$ such that we get the following propagation of freeze.



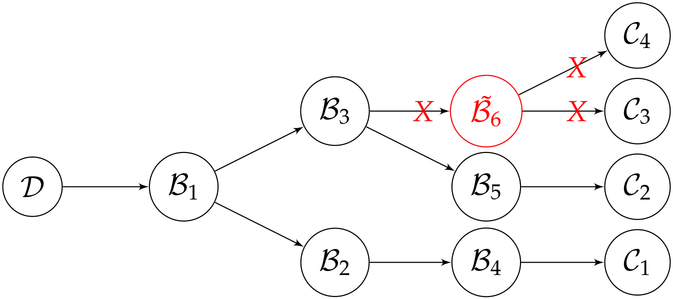
Propagation of Tree Shock (1)

Bank 6 becomes too "risky" (likely to default) and so bank 3 freezes credit to bank 6.



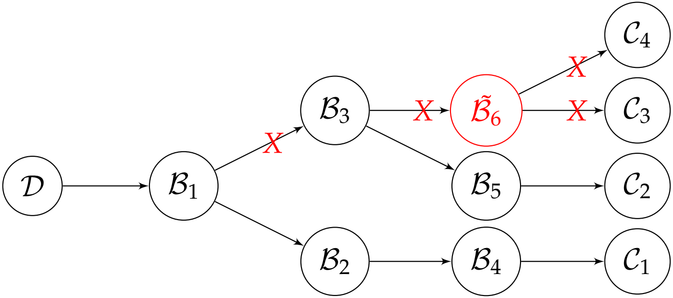
Propagation of Tree Shock (2)

Bank 6 cannot lend to clients 3 and 4 because it does not have access to credit, so freezes their credit in equilibrium.



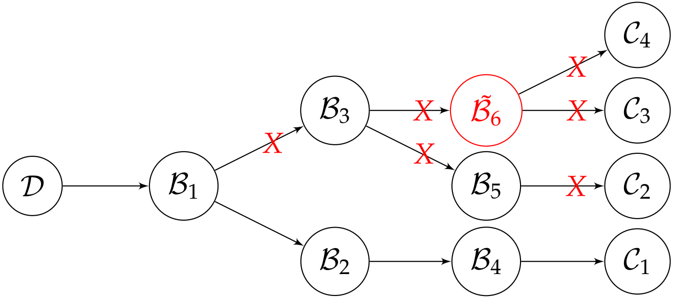
Propagation of Tree Shock (3)

Bank 3 only funds at most one client project now. Bank 1 does not find the investment $\mathcal{B}1 \rightarrow \mathcal{B}3 \rightarrow \mathcal{B}5$ profitable, but found the other two chains profitable enough to still warrant lending. But without this, bank 1 now freezes credit to bank 3.

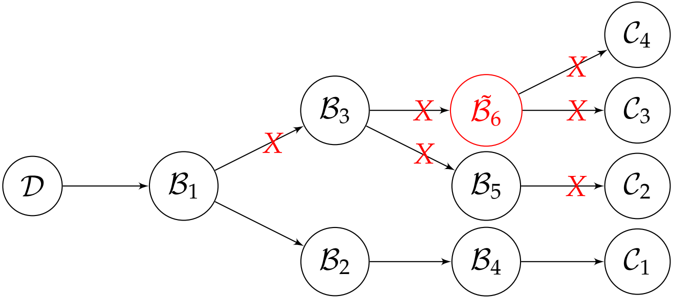


Propagation of Tree Shock (4)

As before, this implies bank 3 must freeze bank 5's credit and bank 5 will freeze client 2's credit, in equilibrium.



Main Finding

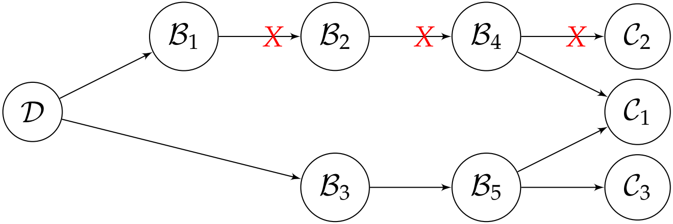


Proposition 3.

If T is a tree, generically there is a unique ex-ante equilibrium (in terms of freezes). Consequently, (i) shocks can only *increase* credit freezes, (ii) the banks affected by the shock occur in a connected cluster, and (iii) within this connected cluster is the bank hit with the shock.

Freezes in General Networks (1)

- Banks compete for contracts; this can affect the profitability of various parts of the network.
- None of (i)-(iii) likely hold for general networks.



Freezes in General Networks (2)

- Competition effects allow client 2 to get funding after a shock to bank 5.
- Formalizing this effect is ongoing work.