Strategic Reviews

James Siderius joint with Mohamed Mostagir
(Preliminary Work)

INFORMS 2019
Seattle, WA, October 22, 2019
Reviews and Influence

Yelp review $25 / $50

We are looking for established Yelp accounts with over 50 reviews (please link Yelp account) to write well-written reviews for a restaurant. Many of these restaurants have a bi-polar review history (mostly positive 4's and 5's but a couple unfiltered 1's dragging them down, either from competitors or disgruntled ex-staff) and need a few 5's to rebuild their rating back. If this is something you'd be interested in, let us know.

The price is a Paypal transfer of $25 for the review, and another $25 to cut and paste that same review onto a couple other social media websites.

- Principals only. Recruiters, please do not contact.
- Please, no phone calls about this.
- Please do not contact job posted.

Posting ID: 3797859002  Posted: 2007-02-15

The price is a Paypal transfer of $25 for the review, and another $25 to cut and paste that same review onto a couple other social media websites.
What We Do

- Consumers want to buy good products. Firms want consumers to buy their product. What role do reviewers play?
  - do reviewers have an incentive to bias reviews?
  - can consumers be influenced by these signals, even with these biases?
  - are firms willing to pay for this persuasion?

- Try to understand strategic incentives between reviewers and firms, and how influence arises endogenously through reputation.

- How can platform limit incentives to accept “bribes,” lie about reviews, and lose influence.

- Today:
  - three-tier model of reviews
  - characterize how reviewers and consumers make decisions
  - investigate how firms can benefit with bribes
What We Do

• Consumers want to buy good products. Firms want consumers to buy *their* product. What role do reviewers play?
  ▶ do reviewers have an incentive to bias reviews?
  ▶ can consumers be influenced by these signals, even with these biases?
  ▶ are firms willing to pay for this persuasion?

• Try to understand *strategic incentives* between reviewers and firms, and how influence arises endogenously through reputation.

• How can platform limit incentives to accept “bribes,” lie about reviews, and lose influence.

• Today:
  ▶ three-tier model of reviews
  ▶ characterize how reviewers and consumers make decisions
  ▶ investigate how firms can benefit with bribes
What We Do

- Consumers want to buy good products. Firms want consumers to buy *their* product. What role do reviewers play?
  - do reviewers have an incentive to bias reviews?
  - can consumers be influenced by these signals, even with these biases?
  - are firms willing to pay for this persuasion?

- Try to understand strategic incentives between reviewers and firms, and how influence arises endogenously through reputation.

- How can platform limit incentives to accept “bribes,” lie about reviews, and lose influence.

- Today:
  - three-tier model of reviews
  - characterize how reviewers and consumers make decisions
  - investigate how firms can benefit with bribes
What We Do

• Consumers want to buy good products. Firms want consumers to buy their product. What role do reviewers play?
  ▶ do reviewers have an incentive to bias reviews?
  ▶ can consumers be influenced by these signals, even with these biases?
  ▶ are firms willing to pay for this persuasion?

• Try to understand strategic incentives between reviewers and firms, and how influence arises endogenously through reputation.

• How can platform limit incentives to accept “bribes,” lie about reviews, and lose influence.

• Today:
  ▶ three-tier model of reviews
  ▶ characterize how reviewers and consumers make decisions
  ▶ investigate how firms can benefit with bribes
Model
Model: Players

(a) **Firms**: Arrive sequentially at each time $t = 1, 2, \ldots$ and live for only one period.

- Each new firm has a quality $q_t$ of its product which is not known (with certainty) to anyone.

(b) **Reviewers**: The same set of agents over time who consume and review each of the products at all times $t$.

- Each reviewer $j$ has a type $\omega_j$ which is either high-skill ($H$) or low-skill ($L$), where she is high-skill with probability $p$.
- High-skill types receive more precise signals of the product quality than do the low-skill types.
- For simplicity, firms and reviewers know all reviewers’ skill types.

(c) **Consumers**: There is a continuum of consumers who have heterogenous preferences for quality. Formally, each consumer has an outside option $\phi_i$ which it can obtain instead of purchasing the product, where:

- $\phi_i$ is increasing in $i$ with $\lim_{i \to 0} \phi_i = -\infty$ and $\lim_{i \to 1} \phi_i = \infty$. 

Model: Players

(a) **Firms:** Arrive sequentially at each time \( t = 1, 2, \ldots \) and live for only one period.
   - Each new firm has a quality \( q_t \) of its product which is not known (with certainty) to anyone.

(b) **Reviewers:** The same set of agents over time who consume and review each of the products at all times \( t \).
   - Each reviewer \( j \) has a type \( \omega_j \) which is either high-skill (\( H \)) or low-skill (\( L \)), where she is high-skill with probability \( p \).
   - High-skill types receive more precise signals of the product quality than do the low-skill types.
   - For simplicity, firms and reviewers know all reviewers’ skill types.

(c) **Consumers:** There is a continuum of consumers who have heterogenous preferences for quality. Formally, each consumer has an outside option \( \phi_i \) which it can obtain instead of purchasing the product, where:
   - \( \phi_i \) is increasing in \( i \) with \( \lim_{i \to 0} \phi_i = -\infty \) and \( \lim_{i \to 1} \phi_i = \infty \).
Model: Players

(a) **Firms**: Arrive sequentially at each time $t = 1, 2, \ldots$ and live for only one period.
   - Each new firm has a quality $q_t$ of its product which is not known (with certainty) to anyone.

(b) **Reviewers**: The same set of agents over time who consume and review each of the products at all times $t$.
   - Each reviewer $j$ has a type $\omega_j$ which is either high-skill ($H$) or low-skill ($L$), where she is high-skill with probability $p$.
   - High-skill types receive more precise signals of the product quality than do the low-skill types.
   - For simplicity, firms and reviewers know all reviewers’ skill types.

(c) **Consumers**: There is a **continuum** of consumers who have heterogeneous preferences for quality. Formally, each consumer has an outside option $\phi_i$ which it can obtain instead of purchasing the product, where:
   - $\phi_i$ is increasing in $i$ with $\lim_{i \to 0} \phi_i = -\infty$ and $\lim_{i \to 1} \phi_i = \infty$. 
Model: Information

- At each time $t$, the firm’s quality and the signals $s_{j,t}$ of each reviewer $t$ are drawn according to the following process:
  - Quality is drawn from the standard normal, $q_t \sim \mathcal{N}(0,1)$.
  - Each reviewer’s signal is an unbiased, noisy signal of the quality $q_t$. High-skill reviewers have less noise around the truth than low-skill reviewers.

- Conditional on $s_{j,t}$, all reviewers $j$ (simultaneously) send reviews $r_{j,t} \in \mathbb{R}$, which are publicly observable to all players.

- Each consumer chooses to either purchase the product ($x_{i,t} = 1$) or not ($x_{i,t} = 0$) at unit price. Consumers receive independent experiences $e_{i,t} = q_t + \eta_{i,t}$, for some noise term $\eta_{i,t}$, where $\eta_{i,t}$ are iid, distributed symmetrically around 0, and have finite variance.
Model: Information

- At each time $t$, the firm’s quality and the signals $s_{j,t}$ of each reviewer $t$ are drawn according to the following process:
  - Quality is drawn from the standard normal, $q_t \sim \mathcal{N}(0,1)$.
  - Each reviewer’s signal is an unbiased, noisy signal of the quality $q_t$. High-skill reviewers have less noise around the truth than low-skill reviewers.

- Conditional on $s_{j,t}$, all reviewers $j$ (simultaneously) send reviews $r_{j,t} \in \mathbb{R}$, which are publicly observable to all players.

- Each consumer chooses to either purchase the product ($x_{i,t} = 1$) or not ($x_{i,t} = 0$) at unit price. Consumers receive independent experiences $e_{i,t} = q_t + \eta_{i,t}$, for some noise term $\eta_{i,t}$, where $\eta_{i,t}$ are iid, distributed symmetrically around 0, and have finite variance.
Model: Information

• At each time $t$, the firm’s quality and the signals $s_{j,t}$ of each reviewer $t$ are drawn according to the following process:
  ▶ Quality is drawn from the standard normal, $q_t \sim \mathcal{N}(0,1)$.
  ▶ Each reviewer’s signal is an unbiased, noisy signal of the quality $q_t$. High-skill reviewers have less noise around the truth than low-skill reviewers.

• Conditional on $s_{j,t}$, all reviewers $j$ (simultaneously) send reviews $r_{j,t} \in \mathbb{R}$, which are publicly observable to all players.

• Each consumer chooses to either purchase the product ($x_{i,t} = 1$) or not ($x_{i,t} = 0$) at unit price. Consumers receive independent experiences $e_{i,t} = q_t + \eta_{i,t}$, for some noise term $\eta_{i,t}$, where $\eta_{i,t}$ are iid, distributed symmetrically around 0, and have finite variance.
Model: Payoffs and Bribes

- **Consumers** are myopic and maximize their current-period utility given posted reviews $r_t$:
  \[
  x_{i,t}^*(r_t) = \arg \max_{x_{i,t} \in \{0,1\}} \mathbb{E}[(e_{i,t} - \phi_i)x_{i,t} | r_t]
  \]
  which has the cutoff strategy $x_{i,t}^*(r_t) = 1$ iff $\mathbb{E}[q_t | r_t] \geq \phi_i$.
  - Let $X_t^*(r_t)$ be the total amount of the product purchased, conditional on $r_t$.
  - We can define the influence index $l_{j,t}$ of reviewer $j$ at time $t$ as:
    \[
    l_{j,t} = \frac{\partial X_t^*(r_t)}{\partial r_{j,t}}
    \]
    - Influence of reviewer $j$ is the sensitivity of a consumer’s decision from $j$’s review.

- Assume **firm** $t$ may offer a bribe schedule, $b_{j,t}(r_{j,t}) \geq 0$, for each reviewer $j$.

- Let $r_t^*$ denote the reviews posted at time $t$. Then the payoff of **firm** $t$ is given by:
  \[
  U_t = X_t^*(r_t) - \sum_{j=1}^{n} b_{j,t}(r_t^*)
  \]
Model: Payoffs and Bribes

- **Consumers** are myopic and maximize their current-period utility given posted reviews $r_t$:
  \[ x_{i,t}^*(r_t) = \arg \max_{x_{i,t} \in \{0,1\}} \mathbb{E}[(e_{i,t} - \phi_i)x_{i,t} | r_t] \]
  which has the cutoff strategy $x_{i,t}^*(r_t) = 1$ iff $\mathbb{E}[q_t | r_t] \geq \phi_i$.
  - Let $X_t^*(r_t)$ be the total amount of the product purchased, conditional on $r_t$.
  - We can define the influence index $I_{j,t}$ of reviewer $j$ at time $t$ as:
    \[ I_{j,t} = \frac{\partial X_t^*(r_t)}{\partial r_{j,t}} \]
  - Influence of reviewer $j$ is the sensitivity of a consumer’s decision from $j$’s review.

- Assume **firm** $t$ may offer a **bribe schedule**, $b_{j,t}(r_{j,t}) \geq 0$, for each reviewer $j$.

- Let $r_t^*$ denote the reviews posted at time $t$. Then the payoff of **firm** $t$ is given by:
  \[ U_t = X_t^*(r_t) - \sum_{j=1}^n b_{j,t}(r_t^*) \]
Model: Payoffs and Bribes

- **Consumers** are myopic and maximize their current-period utility given posted reviews $r_t$:
  \[ x^*_{i,t}(r_t) = \arg \max_{x_{i,t} \in \{0,1\}} \mathbb{E}[(e_{i,t} - \phi_i)x_{i,t}|r_t] \]
  which has the cutoff strategy $x^*_{i,t}(r_t) = 1$ iff $\mathbb{E}[q_t|r_t] \geq \phi_i$.

  - Let $X^*_t(r_t)$ be the total amount of the product purchased, conditional on $r_t$.
  - We can define the influence index $I_{j,t}$ of reviewer $j$ at time $t$ as:
    \[ I_{j,t} = \frac{\partial X^*_t(r_t)}{\partial r_{j,t}} \]
    Influence of reviewer $j$ is the sensitivity of a consumer’s decision from $j$’s review.

- Assume **firm** $t$ may offer a bribe schedule, $b_{j,t}(r_{j,t}) \geq 0$, for each reviewer $j$.

- Let $r^*_t$ denote the reviews posted at time $t$. Then the payoff of **firm** $t$ is given by:
  \[ U_t = X^*_t(r_t) - \sum_{j=1}^{n} b_{j,t}(r^*_t) \]
Some reviewers are behavioral and always truthfully report (with small probability $\epsilon > 0$) (i.e., $r_{j,t} = s_{j,t}$). Otherwise, the reviewer is strategic.

Assume reviewers also care about their influence over consumers’ decisions according to propensity $\beta > 0$ (e.g., status or preferential treatment).

Strategic reviewers are infinitely patient and maximize their average payoff:

$$V_j = \sum_{t=0}^{\infty} \delta^t (\beta_j l_{j,t} + b_{j,t})$$

where we have suppressed the RHS dependence on the history of reviews.

Classify “pure-strategy” perfect Bayesian equilibria as $\delta \to 1$ (where reviewers are infinitely patient).
• Some reviewers are behavioral and always truthfully report (with small probability \( \epsilon > 0 \)) (i.e., \( r_{j,t} = s_{j,t} \)). Otherwise, the reviewer is strategic.

• Assume reviewers also care about their influence over consumers’ decisions according to propensity \( \beta > 0 \) (e.g., status or preferential treatment).

• Strategic reviewers are infinitely patient and maximize their average payoff:

\[
V_j = \sum_{t=0}^{\infty} \delta^t (\beta_j l_{j,t} + b_{j,t})
\]

where we have suppressed the RHS dependence on the history of reviews.

• Classify “pure-strategy” perfect Bayesian equilibria as \( \delta \to 1 \) (where reviewers are infinitely patient).
• Some reviewers are behavioral and always truthfully report (with small probability \(\epsilon > 0\)) (i.e., \(r_{j,t} = s_{j,t}\)). Otherwise, the reviewer is strategic.

• Assume reviewers also care about their influence over consumers’ decisions according to propensity \(\beta > 0\) (e.g., status or preferential treatment).

• Strategic reviewers are infinitely patient and maximize their average payoff:

\[
V_j = \sum_{t=0}^{\infty} \delta^t (\beta_j l_{j,t} + b_{j,t})
\]

where we have suppressed the RHS dependence on the history of reviews.

• Classify “pure-strategy” perfect Bayesian equilibria as \(\delta \to 1\) (where reviewers are infinitely patient).
Some reviewers are behavioral and always truthfully report (with small probability $\epsilon > 0$) (i.e., $r_{j,t} = s_{j,t}$). Otherwise, the reviewer is strategic.

Assume reviewers also care about their influence over consumers’ decisions according to propensity $\beta > 0$ (e.g., status or preferential treatment).

Strategic reviewers are infinitely patient and maximize their average payoff:

$$V_j = \sum_{t=0}^{\infty} \delta^t (\beta_j l_{j,t} + b_{j,t})$$

where we have suppressed the RHS dependence on the history of reviews.

Classify “pure-strategy” perfect Bayesian equilibria as $\delta \to 1$ (where reviewers are infinitely patient).
For each time $t$:

Try to understand equilibrium behavior for all $t > T$, for some large $T$ (referred to as “eventually”).
Babbling-Trigger Equilibrium

- **Babbling equilibrium:** Consumers ignore the reviews and instead choose $X_t^*$ according to their prior, $X_t^* = \phi^{-1}(0)$.
  - **On-path play:** Reviewers abstain or send defunct signals and consumers do not listen.
  - **Off-path play:** Consumers assume any posted reviews are spurious and do not reflect true quality.

- **When is babbling an equilibrium?**
  - Because small probability $\epsilon > 0$ the reviewer will post $r_{j,t} = s_{j,t}$ always, babbling forever is not a best-response from the beginning (for a consumer).
  - Once a reviewer has been determined to not be truthtelling type (with sufficiently high probability), consumers credibly can switch to babbling.

- **Babbling-trigger** represents a harsh consumer who punishes reviewers who, beyond a reasonable doubt, can be identified as untruthful in their reviews.
  - This equilibrium provides a best-case scenario for the efficacy of the platform.
Reviewer-Consumer Reputation Game

- Assume firms do not offer any bribes; that is, the bribe schedules are given exactly by \( b_{j,t}(r_{j,t}) \equiv 0 \) for all \( j, t \).

**Theorem**

In the babbling-trigger equilibrium, all reviewers are eventually honest (i.e., \( r_{j,t} = s_{j,t} \)) and consumers eventually infer the true types of every reviewer.

- Consumers eventually use the inverse-variance weighted average to infer expected quality:

\[
\mathbb{E}[q_t|r_t] = \frac{\sum_{j=1}^n r_{j,t}/\sigma_{j,t}^2}{1 + \sum_{j=1}^n 1/\sigma_{j,t}^2}
\]

and where \( X_t^*(r_t) = \phi^{-1}(\mathbb{E}[q_t|r_t]) \). Influence index is higher for high-skill types.

- **Intuition:** Suppose I’m a low-skill type \( \sigma_L = 100 \) and want to match high-skill type \( \sigma_H = 1 \).
  - Correlation between my \( s_t \) and \( q_t \) will be 1/101 instead of 1/2.
  - Can I improve my correlation by biasing my \( s_t \)?
Fixed Bribe Schedules

- Suppose we take the bribe schedules $b_{j,t}(r_{j,t})$ as given but not necessarily equal to zero. How does the equilibrium change?

**Theorem**

*When bribe schedules are fixed, in the babbling-trigger equilibrium every low-skill type reviewer eventually reports truthfully ($r_{j,t} = s_{j,t}$). On the other hand, every high-skill reviewer eventually either: (i) reports truthfully ($r_{j,t} = s_{j,t}$) or (ii) plays a strategy where $r_{j,t} = s_{j,t} + \varepsilon'_{j,t}$ with $\mathbb{E}[\varepsilon'_{j,t}] = 0$, $\mathbb{E}[(\varepsilon'_{j,t})^2] = \sigma_L^2 - \sigma_H^2$, and $\varepsilon'_{j,t} \perp s_{j,t}$.*

**Key Takeaway:** High-skill reviewer can choose to mimic a low-skill reviewer. The reviewer accepts the bribe, and pretends to “inject noise” into his review but instead biases her signal.

- Tradeoff between bribe payment and influence: high-skill reviewer will have the influence of a low-skill reviewer, but receive payments.
- Consumers still make purchase decisions in the exact same way!
- But, valuable information from reviewer to consumer destroyed.
Strategic Firms

• Bribe schedules are endogenous decisions on the part of the firm.
  ▶ Reputation game between firm and reviewer as well.
  ▶ Reviewer’s influence has more than just intrinsic value, as influence translates into payoffs for firm.

• For example, take $\beta$ to be very small (i.e., little intrinsic value derived for influence). Should the reviewer accept large bribes in exchange for very biased reviews?
  ▶ No, in the babbling-trigger equilibrium, eventually this reviewer will lose all influence. No future firms will offer bribes.

• Each entering firm must choose $b_1(r_1), \cdots, b_n(r_n)$, then reviewers observe signals, and post reviews.
  ▶ Look for a stationary equilibrium where all firms post the same bribe schedules and high-skill reviewers decide whether to report truthfully or mimic low-skill.
  ▶ Consumers still continue to use inverse-variance weights to make purchase decisions.
Assume there are just two reviewers both with high-precision. How does bribe schedule $b_1(r_1)$ affect the decisions of reviewers 1 and 2 to mimic low-precision?

- Clearly, increasing the slope of $b_1(r_1)$ will increase incentives for reviewer 1 to mimic imprecision.
- But, an ambiguous effect on reviewer 2...

![Diagram showing the effects of bribes on substitutive and complementary interactions.](image-url)
Platform Incentives and Ongoing Work

- Characterization of firm’s optimal bribe schedule, given reputation game between reviewers and consumer.

- **Optimal policy**: Can the platform reward influence (and push up $\beta$ for some reviewers)?
  - Decrease incentives to mimic low-skill and instead report truthfully.
  - **Externalities**: possible my decision to report truthfully can nudge others to do the same.
  - Relatively inexpensive revenue-sharing can restore substantial amounts of information on the platform.